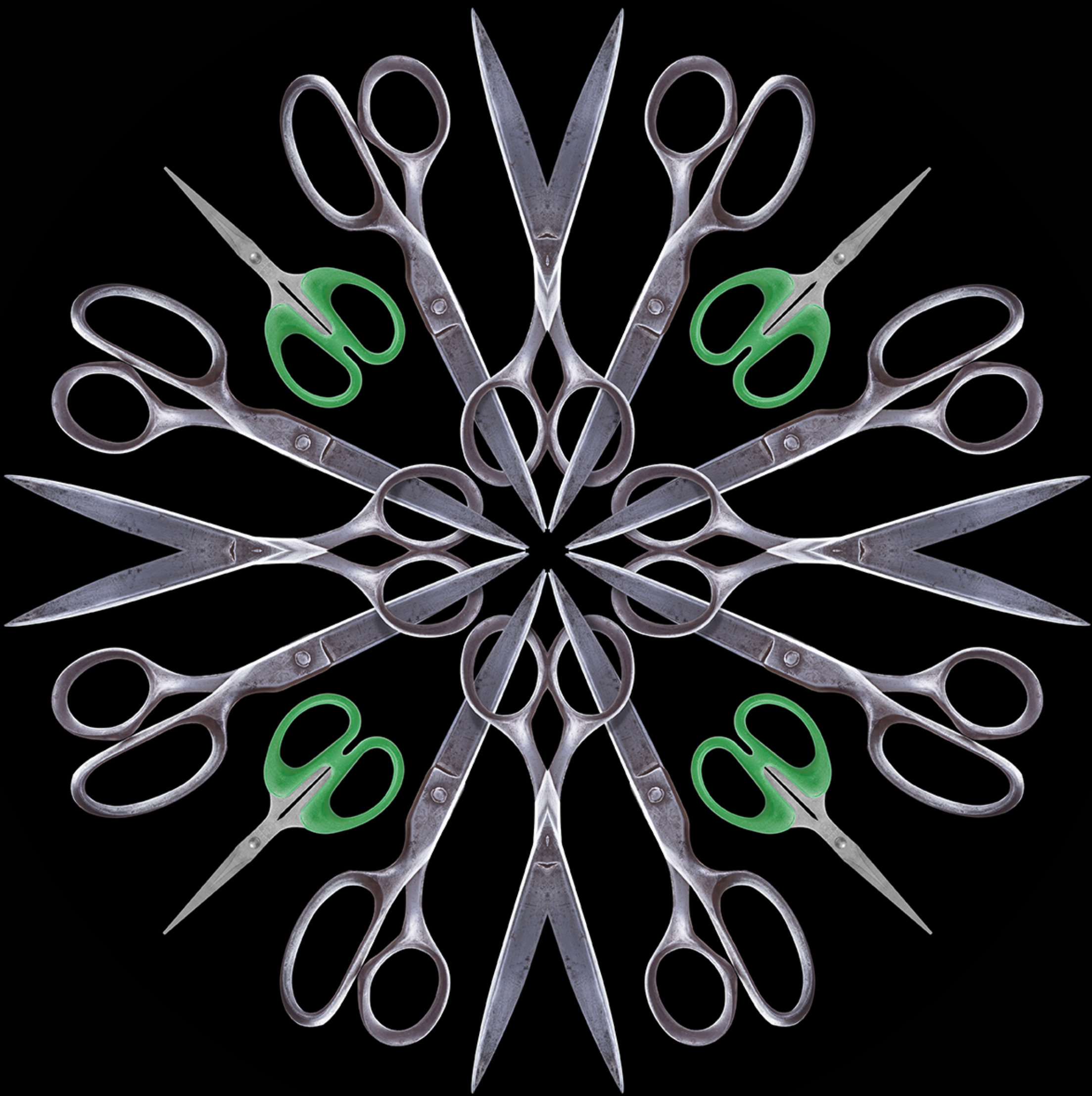


FRTB for structured products
The impact of an internal model under FRTB

Financial Risk

The impact of an internal model under FRTB

What is the value of an internal model approach in the structured products business?

A quantitative analysis comparing capital requirements under a standardised approach versus an internal model.

Contents

1. Introduction

The complexity of measuring risks in a structured products portfolio

Structured products represent an important investment class in Switzerland. The total "live" volume of structured products is roughly CHF 200bn[1], which represents 3% of all securities held in custody accounts in Switzerland, with the remainder consisting of direct investments mainly in bonds, shares, and funds [8]. A large share of outstanding structured products in Switzerland are yield-enhancement products such as barrier reverse convertibles. Such products typically have path-dependent pay-offs, which cannot be priced using closed-form formulas. Given that the pricing of individual issuances already requires sophisticated approaches (often including Monte Carlo simulations), there are substantial challenges to the measurement of portfolio-level risks with high accuracy.

Capital charges and the Fundamental Review of the Trading Book.

Structured products and of Businesses involving non-vanilla instruments like the structured products business typically attract significantly higher regulatory capital requirements than vanilla flow businesses. Even when hedged closely, the non-linearity of the payoffs leads to residual risks that attract punitive regulatory capital charges under the Basel standardised approach for market risk, which are expected even to increase substantially under the revised standardised approach of the Fundamental Review of the Trading Book (FRTB).

The new market risk framework FRTB does not only represent an overhaul of the standardised approach, it also introduces new standards to be met under the Internal Model Approach (IMA). Our analyses show that for structured products portfolios the IMA can reduce capital charges by up to 70%[2] compared to the revised standardised approach. An IMA, however, is accompanied by a range of model acceptance criteria, which serve as prerequisites for obtaining and retaining IMA model approval.

Key challenges for Internal Model Approaches

One of the key model acceptance criteria is regular P&L attribution testing, which focuses on closely aligning the risk pricing with front office pricing. This testing creates significant challenges for obtaining model approval of structured products businesses, yet failure to comply with the testing will result in the application of the punitive FRTB standardised approach. A full revaluation approach in the IMA ES modelling would minimize the risk of failing the P&L attribution testing. In the context of the structured products business, however, such an approach would require a nested simulation, which is commonly considered to be computationally unachievable.

Our solution

In this article we set out an innovative expected shortfall modelling approach. It enables full revaluation of a structured products portfolio within a Monte Carlo framework (nested Monte Carlo), but is, however, still computationally achievable. While this approach has been implemented for a structured products business, it is suitable for integration into a wider full portfolio modelling framework.

It is time to assess the capital impact of an Internal Model Approach on your structured product portfolio.

1. A historical overview of the Swiss Structured products market is presented in Appendix I.
2. Actual reduction is subject to the 72.5% output floor, which is applied to total Pilla1 capital requirements, including credit risk, operational risk and non-counterparty related assets

2. Setting the Scene

Addressing the shortcomings in previous market risk capital charges

The Basel Committee has undertaken a fundamental overhaul of the market risk framework to address significant weaknesses leading to an undercapitalisation of certain trading activities prior to the 2007-2008 credit crisis. It published the new rules, referred to as the fundamental review or the trading book (FRTB), in January 2016. Building on changes to the market risk framework introduced through Basel 2.5 in 2009, it requires banks under the IMA to enhance their models' risk capture, and to implement a new standardized approach (SA), which is the basis for the Basel 3 output floor.

Change in Internal Modal Approach methodology: expected shortfall

FRTB represents a significant evolution of market risk methodology. Key changes include the requirement to measure tail risk using expected shortfall (ES) instead of value at risk, and to use varying liquidity horizons, replacing the uniform 10 days under Basel 2.5. These changes aim to capture tail losses during periods of significant financial stress in a more prudent way, and to measure risks associated with a sudden and significant deterioration of liquidity across different markets.

Strict acceptance criteria for IMA approval

However, besides the changes in methodology, FRTB also imposes a much stricter model approval and on-going model governance process. The enhanced acceptance criteria have been criticised by the industry as they have significantly raised the bar for applying an IAM broadly across trading activities.

Banks are expecting challenges in relation to the FRTB governance process, comprising **regular-back testing** and **P&L attribution testing**, since they have to perform the tests **at trading desk level**. If a bank exceeds certain quantitative thresholds associated with the tests, it has to move the trading desk for which the breach occurred under the SA on a permanent basis.

The tests involve a comparison of the risk measure and P&L values generated by the model against the daily P&L obtained using front office pricing models. Banks must assess the acceptance criteria at the trading desk level, as opposed to the full portfolio. Hence, they have to capture within the IMA more intricate risks associated with non vanilla instruments that previously would have been insignificant to a bank's overall trading portfolio.

P&L attribution testing – a key challenge for structured product businesses

For non-vanilla instruments, the industry often makes use of approximations and shortcuts, assuming that the higher order risks will be "lost in the noise" of the rest of the portfolio. Banks are hence concerned that, under the new FRTB rules, certain books containing non-vanilla instruments cannot be included in the IMA scope, since it will be difficult to meet the IMA acceptance criteria on an ongoing basis.

The FRTB framework includes a revised SA for measuring market risk that aims at increasing the risk sensitivity and risk capture. Given the importance of the new SA in the context of the Basel 3 internal models output floor, regulators considered the changes to be a prerequisite in order to increase the credibility of the SA as a methodology applied to large complex trading portfolios.

Consequences of not meeting acceptance criteria

At the core of the FRTB SA, there is a sensitivity-based approach (SBA) for estimating market risk tail losses. For certain non-vanilla instruments, however, additional conservative add-ons over and above the SBA charge apply.

For IMA banks, having a desk carved out from the model, will typically not only lead to significantly higher capital charges for that particular desk, it will also lead to increased operational difficulties when it comes to internal risk transfers, as well as a decrease in risk diversification across the desks that are covered by the internal model.

3. Structured products under FRTB: a first glance

Benchmark for barrier reverse convertible

Given the current low interest rate environment, the most popular structured product category is the yield enhancement products. The typical yield enhancement product, which we used for the analyses within this document, is a barrier reverse convertible referencing one or more underlying equities. In a barrier reverse convertible, the investor receives a coupon throughout the product's lifetime, but carries the potential downside of the worst performing stock at maturity (similar to a short put option). The issuers of structured products are thus holding a "short" position in the underlying equities and will have a positive curvature or "gamma" (similar to a long put option).

The barrier reverse convertible is a "yield enhancement" product, as the coupon includes a premium, which compensates for the potential downside. Appendix II gives a more detailed description of the payoff for a range of types of barrier reverse convertibles.

Institutions typically delta hedge their structured products portfolio. Assuming that the hedge consists mainly of cash positions in the underlying equities, the issuer will hold a positive curvature position[3]. Hence, both an increase and a decrease in the underlying stock leads to a mark-to-market gain of the delta-hedged portfolio. The key risk run by a delta-hedged structure product business is hence vega risk, more specifically, a drop in market volatility can lead to significant market value losses.

Key capital contributions under FRTB

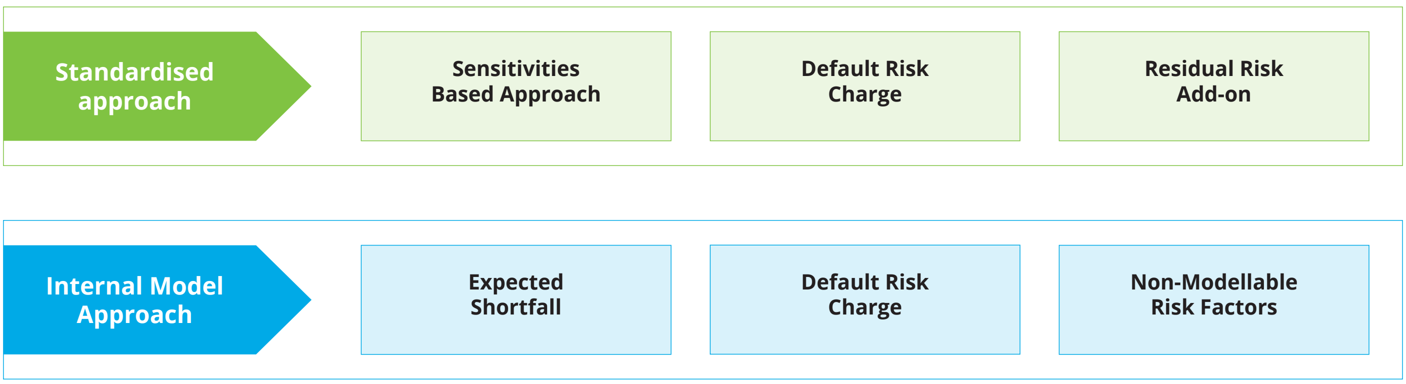
In this article, we assess the impact of the FRTB on the market risk capital requirements for both SBA and IMA for a sample portfolio of barrier reverse convertibles (see Appendix III).

Components of the FRTB standardised approach

The new standardised framework consists of three components: the SBA, the default risk charge and the residual risk add-on. The SBA makes use of the delta, vega and gamma of the different trades to determine a market risk capital charge. The new default risk charge aims to capture the losses on the trading portfolio due issuers of equities and bonds defaulting. Finally, the residual risk add-on is a conservative notional value based add-on mainly for instruments with a non-linear pay-off that cannot be replicated with vanilla options. It aims to capture market risks beyond those captured by the SBA.

Components of an IMA under FRTB

As under the SA, there are also three components to the IMA capital charge: the ES calculation, the default risk charge, and the stressed capital add-on. The scope of the ES calculation is broadly similar to the SBA. However, it aims to capture price sensitivities beyond delta, vega and gamma, in line with front office pricing. The default risk charge has the same purpose for the IMA as under the SA. Finally, the stressed capital add-on provides an additional charge for non-modellable risk factors.



3. Typically, issuers hedge would their structured products portfolios with a combination of cash equities and exchange traded options. We, however, expect that including some exchange traded options in the hedge portfolio will not change the direction and significance of our results.

Key risk components for structured products
In our analyses, we look into structured products from an issuer's perspective. For an unhedged portfolio of barrier reverse convertibles, this means that the institution is short in the underlying equities, so that a default in the equities leads to a P&L gain. Issuers typically delta-hedge by buying the underlying stock or exchange traded options. For our analyses, we assume the hedge portfolio to consist of cash equities. In this case, the positive curvature of the portfolio would mean that a sudden default of the equity issuer would still lead to a windfall gain. Hence, in both cases, no default risk charge applies.

Structured products reference in most cases names that are contained in the major indices of the major stock markets. We hence assume that no requirements for non modellable risk factors apply.

This residual risk add-on can be especially punitive for high notional low risk trades, which are more prevalent in interest rate and FX trading than equities trading. For our portfolio of barrier reverse convertibles, however, we consider this add-on to be less material. In what follows, we show that the IMA leads to lower capital charges when comparing the SBA versus the IMA.

In the remainder of the article, we hence focus on comparing the SBA to the IMA.

Standardised Approach
A floor in the gamma risk calculation in the SBA, ensures that the benefits gained from the positive curvature cannot be taken into account. Given the portfolio's positive curvature, we expect a zero gamma risk contribution under the SBA.

In absence of delta risk (delta-hedged portfolio), we expect the SBA to be completely driven by the vega risk component. We hence expect the direction and significance of the SBA market risk contributions to be as follows:

Portfolio	Delta Risk	Vega Risk	Gamma Risk	Total SBA
Unhedged	++	+	0	+++
Delta-hedged	0	+	0	+

At first glance, one might think that, due to the zero gamma risk contribution, the SBA is rather favourable, especially for delta-hedged portfolios[4]. Could it potentially even be less punitive than the IMA?

4. Typically, issuers hedge would their structured products portfolios with a combination of cash equities and exchange traded options. We, however, expect that including some exchange traded options in the hedge portfolio will not change the direction and significance of our results.

Internal Model Approach

Under an IMA, the bank simulates the joint future realisations of the portfolio's risk factors, and calculates the P&L associated with each scenario. This then yields a P&L distribution, from which the bank obtains the 97.5th expected shortfall. An IMA approach captures in a consistent way the dependencies between delta, vega, gamma and other sensitivities.

The SBA, on the other hand, calculates a separate risk charge for delta, vega and gamma, without taking into account their dependencies. One typically has that the delta, vega and gamma shocks under the SBA, as well as the aggregation of the shocks, is more conservative than one would expect in an internal model. Furthermore, and particularly relevant to our portfolio of barrier reverse convertibles, the SBA applies a floor to the gamma risk component, which means the gamma offset is not captured.

Given this, one would hence expect the IMA to yield a lower capital charge. This was also observed in a BCBS Quantitative Impact Study, which estimates that market risk capital charges under the revised standardised approach are approximately[5] 4 times larger than those under the revised internal models approach (i.e., 40% higher) for the median bank.

The risk offsets generated by a positive curvature are captured in an internal model.

5. As a comparison: the gamma contribution for a long portfolio of yield enhancement products can be similar in size to the delta risk contribution.

4. Modelling Considerations

Risk factor selection

In a first step, the modeller has to identify the risk factors relevant for the valuation of the portfolio. Whilst many risk factors are obvious choices, some more intricate risk factors will require the modeller to strike a balance between revaluation accuracy and computational performance.

Risk factor generation:

Once the set of risk factors has been identified, an approach has to be devised in which one can generate their future joint realisations. The method of choice is here is often a Monte Carlo simulation.

The main difficulties faced in this step are related to calibration. The modeller has to calibrate the marginal distributions of each individual risk factor and in addition determine the correct joint behaviour (copula) between them. Section 6.2 briefly discusses potential different risk factor modelling approaches.

Portfolio revaluation

Under each future realisation of the risk factors, one has to calculate the portfolio loss. This requires a revaluation of all the products within the portfolio, conditional on the generated values that are associated with the set of risk factors.

Given that a Monte Carlo simulation would typically generate several tens of thousands of paths, each portfolio revaluation needs to be performed within a fraction of a second. This becomes particularly tricky when dealing with exotic derivatives (e.g. barrier reverse convertibles). Indeed, typically the valuation of exotic products does not have a closed form formula, and thus requires a Monte Carlo simulation in itself. Running a separate Monte Carlo for each individual exotic derivative within the portfolio would take far longer than a fraction of a second, even using today's fastest computers.

The industry avoids the nested Monte Carlo by approximating the market value of exotic options using sensitivities or other shortcuts such as pricing grids. Approximation approaches, however, are not able to completely capture the changes in price that are captured in the front office pricing, and are likely to lead to issues when it comes to P&L attribution testing. In this paper, however, we introduce a way to apply a full front office revaluation in the internal model, whilst still remaining computationally feasible. This is achieved by reducing the valuation of the entire portfolio to a single Monte Carlo simulation, as opposed to a Monte Carlo for each individual trade. The approach is discussed further in Section 5.2.

5. Outline of Internal Model Approach

Introduction

Under the FRTB Internal Model Approach, banks will have the flexibility in designing their internal model albeit respecting the minimum standards as outlined in [2]. The risk capital charge under FRTB IMA is based on an ES at the 97.5th quantile at varying liquidity horizons (depending on the underlying risk factor).

The basis of the IMA is a portfolio-level 97.5% ES calibrated to a 10-day risk horizon, whereby all risk factors are shocked. In addition, the IMA requires an ES calculation (also at a 10-day horizon) for different risk-factor subsets, depending on their prescribed liquidity horizons. The 10-day ES for the different liquidity subsets is then rescaled to the longer horizons. The regulatory liquidity-adjusted ES is then given by:

$$ES = \sqrt{\left(ES_{10d}(P)\right)^2 + \sum_{j\geq 2}\left(ES_{10d}(P,j)\sqrt{\frac{\left(LH_j - LH_{j-1}\right)}{10}}\right)^2},$$

where $ES_{10d}(P)$ denotes the 10-day ES for all portfolio risk factors P, and $ES_{10d}(P,j)$ denotes the 10-day ES of the portfolio for the risk-factors with liquidity horizon greater than or equal to LH_j.

The ES measure must be calibrated to a period of significant stress. This period corresponds to the 12-month period within the observation horizon (spanning back to at least 2007) in which the bank's portfolio would have experienced the largest loss. Further details can be found in [2].

Nested Monte Carlo approach

We have devised a cutting-edge Internal Model approach allowing a full revaluation of any exotic equity derivatives.

Classical portfolio derivative pricers typically value each individual derivative separately, which can be computationally expensive for a portfolio of many exotic derivatives. Instead of simulating the future realisations of the underlyings for each individual option, our approach simulates the future realisations of all the underlyings in the portfolio together and determines for each scenario the payoff for every option in the portfolio simultaneously. This approach is significantly faster for a portfolio with a limited number of underlyings, which is typically the case for an issuer of structured products.

Our portfolio valuation approach scales with the number of underlyings in the structured product portfolio and not with the number of products. This hence **enables the application of a nested Monte Carlo.**

Our prototype is parallelised and can run on different cores and servers, leading to a considerable reduction of the simulation time. The approach can be scaled to structured products portfolios of the leading issuing institutions, and can be easily incorporated as a sub-module within a wider IMA.

The nested Monte Carlo approach enables consistency between front office pricing and the internal model, and is hence substantially mitigates the risk of failing **P&L attribution testing.**

Our approach is summarised in Figure 1, and consists of the following key steps:

1. The current portfolio value (i.e., at time t=0) is calculated by means of a single Monte Carlo simulation, the **"pricing Monte Carlo"**. The pricing Monte Carlo is **market implied** (i.e., risk-neutral), and hence makes use of the typical derivative pricing models to determine the realisations of the stock-prices such as Black Scholes, Heston, etc.

2. A Monte Carlo simulation, the **"Risk-Factor Monte Carlo"**, is run to determine future realisations of the portfolio risk factors. In the case of our portfolio, the simulated risk factors are the underlying equity prices and their volatilities. The risk-factor generation is a **"Physical"** (i.e., real-world) simulation, and hence often calibrated to historical risk-factor realisations.

3. For each realisation of the set of risk factors determined in Step 2 (i.e., for each scenario of the risk-factor Monte Carlo), we revalue the portfolio using the same approach outlined in Step 1 (the market-implied pricing Monte Carlo), using the new simulated risk factors as "initial condition".

For the pricing Monte Carlo used in Steps 1 and 3, the quantity of interest is the mean across the different paths, and therefore one does not require that many simulations to achieve numerical stability. However, the risk factor simulation in Step 2 aims at estimating tail losses, and hence requires more simulations to yield a stable estimate.

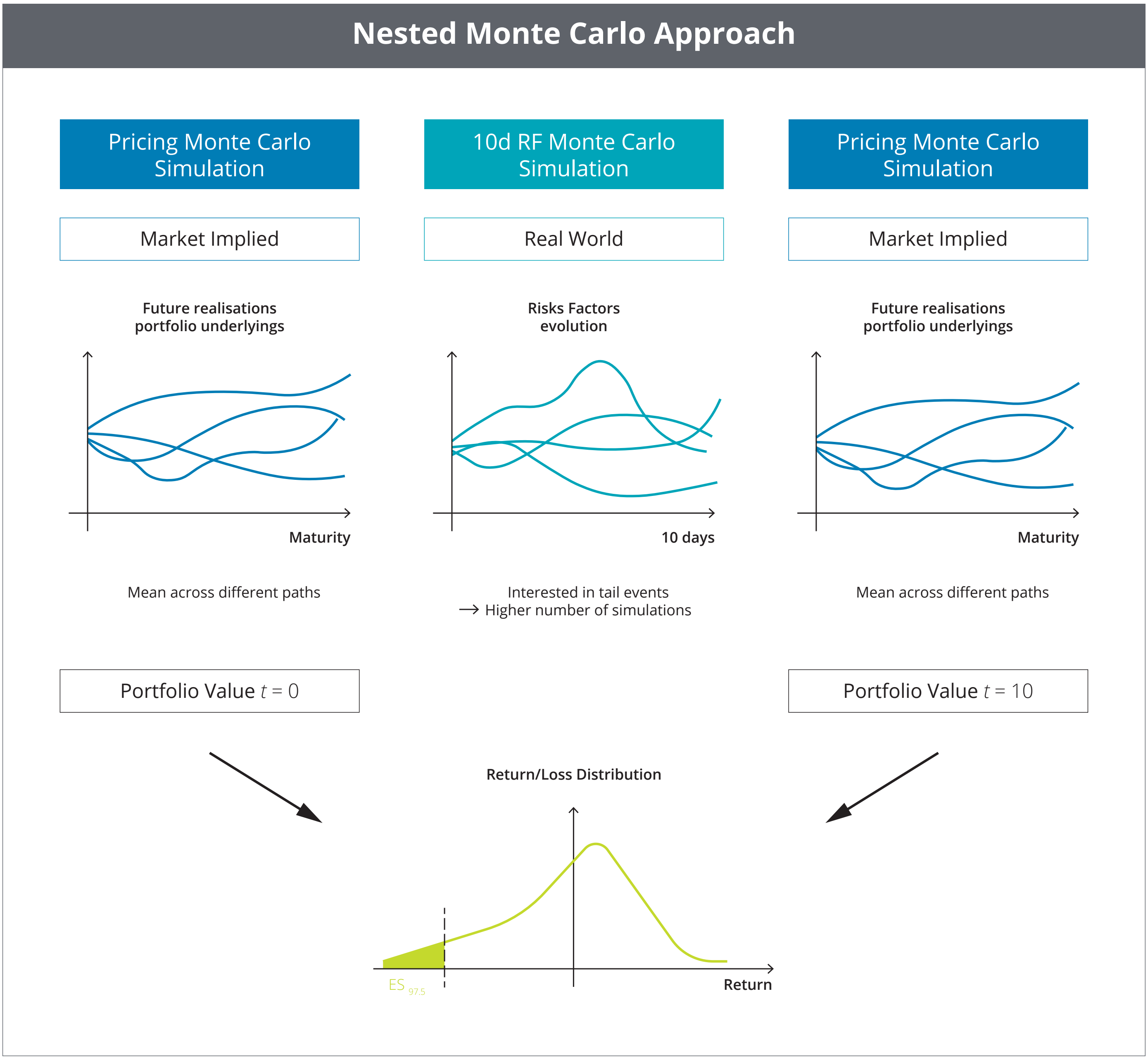


Figure 1 Internal Model Approach using Nested Monte Carlo

6. Results

In this section, we provide a quantitative comparison between risk capital charges under the Basel 2.5 standardised approach[6], the SBA and the ES or the FRTB standardised and internal model approach, respectively. We assess the impacts for a portfolio of short autocallable barrier reverse convertibles that we define in Appendix III.

We observe that banks under the standardised approach would incur a significant increase of roughly 129% when transitioning from Basel 2.5 to Basel 3. Applying an IMA under Basel 3, would reduce capital requirements by 61%, leading to capital levels below the Basel 2.5 standardised approach.

Overview of Results

The market value of our unhedged sample portfolio is 16mioCHF[7] (liability). Due to the negative delta of a short barrier reverse convertible, a delta-hedged portfolio requires the purchase of 10mioCHF in the underlying stocks, yielding a hedged portfolio value of 6mioCHF (net liability).

Delta-hedged portfolio

Both the Basel 2.5 standardised approach and the FRTB SA allow for a complete delta risk reduction. For the hedged portfolio, this means that the risk capital charge under these approaches stems completely from the vega risk component (given the positive gamma).

The market risk capital charge more than doubles under the new standardised approach.

An Internal Model can reduce risk capital requirement for a hedged structured products portfolio by more than 60%.

Figure 2 summarises the risk capital charges under these three different approaches. This graph illustrates our assertions made in Section 3.2.2, namely that an internal model yields significantly lower market risk capital charges. A key contributor for our particular portfolio is the fact that the internal model captures the benefit of the negative gamma/curvature, while both the standardised FRTB and Basel 2.5 approaches do not.

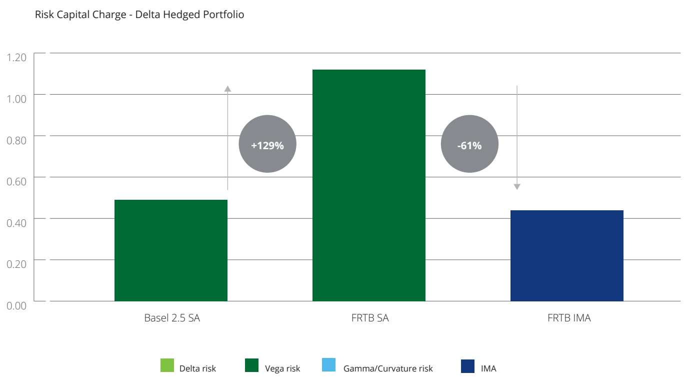


Figure 2 Risk Capital Charge for delta-hedged portfolio

Unhedged portfolio

The unhedged portfolio has positive vega, negative delta and positive gamma/curvature sensitivities. Similarly to the hedged portfolio, the risk charge more than doubles when going from the Basel 2.5 to the FRTB standardised approach. The increase is mainly due to the much higher risk weights applied to sensitivities in the FRTB SA.[8]

Figure 3 summarises the risk capital charges for the unhedged portfolio under these three different approaches. Once again, the graph illustrates nicely that, also for the unhedged portfolio, an internal model can significantly reduce market risk capital. We observe that, under Basel III, the IMA leads to reduction of 68% in capital compared to the SA.

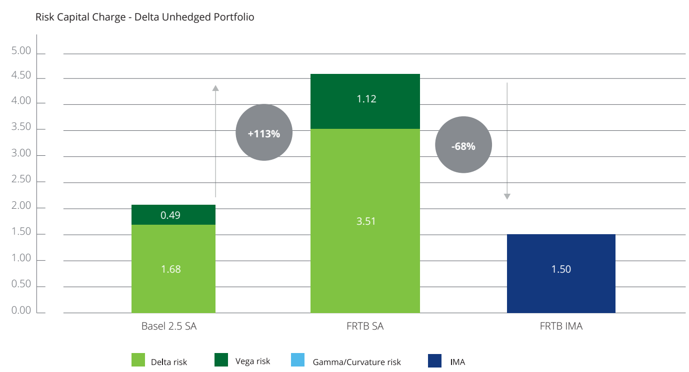


Figure 3 Risk Capital Charge for the unhedged portfolio

6. Basel 2.5 Delta Plus approach
7. The valuation assumes a simple (correlated) Black-Scholes process for the underlying stock prices (with flat equity volatilities)
8. Further details for the calculation, see Appendix B.

Impact of different modelling assumptions

In order to cover different stochastic simulation approaches, we consider five different stochastic methods to simulate the 10-day risk factor evolution:

- driftless Brownian Motion,
- Brownian Motion with drift,
- Historical Sampling,
- t-Copula with empirical historical marginal distributions
- Exponentially Weighted Moving Average.

Appendix IV summarises the five stochastic process models used to simulate the 10 days risk factor evolution.

All five approaches were calibrated to a stressed scenario, as required by the regulation. In our case, the parameters were calibrated using daily historical returns during the 2007-2009 crisis.

The results in Figure 4 below show that changing the distribution assumption does not change the change the significance of the capital reduction.

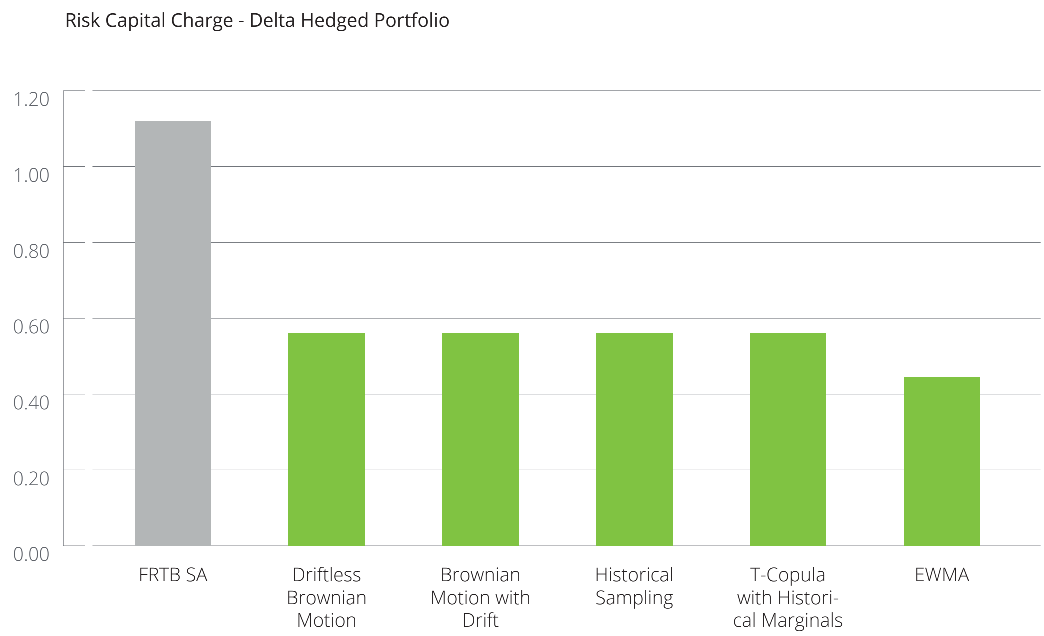


Figure 4 Risk Capital Charge in mioCHF under different stochastic models for the delta-hedged portfolio

7. Conclusion

FRTB acceptance criteria affecting the internal model scope

The new FRTB framework's stringent acceptance criteria are having an impact on what financial institutions are including into their internal model scope, and is even making institutions consider not applying an internal model for regulatory purposes all together. These considerations might be particularly the case for regulated institutions with a significant structured products business.

P&L attribution testing introduces modelling challenges

One of the key drivers behind the change in attitude towards an IMA is the perceived challenges regarding the daily P&L attribution testing at the desk level. Nowadays, financial institutions with internal model approaches often make use of approximations and shortcuts when pricing non-vanilla derivatives. The P&L attribution tests, however, will require banks to more accurately capture their front office pricing models within their internal model calculations.

A smart nested Monte-Carlo approach can help tackle P&L attribution testing

This article introduces a cutting-edge IMA allowing for a full revaluation of exotic equity derivatives. This full revaluation enables consistency between front office pricing and the internal model, and significantly mitigates the risk of failing P&L attribution testing.

Our prototype allows for a full revaluation of any exotic equity derivatives, and is scalable to real-life structured product portfolios. Furthermore, the approach can be seamlessly incorporated as a sub-module within a bank's wider IMA.

Impact of IMA on market risk capital

One of the key benefits of an IMA is the ability to more accurately capture a portfolio's actual markets risks. The alternative, the standardised approach, is a "one size fits all" approach and is expected to yield higher capital charges.

This paper assesses the market risk capital charge under different approaches (standardised and IMA) for a sample portfolio of barrier reverse convertibles. Our analyses confirm the observations from previous Quantitative Impact Studies conducted by the Basel Committee [4] that the market risk capital charges will significantly increase (more than double) under the new standardised framework. Whereas Quantitative Impact Studies indicate that internal models yield on average 30% lower capital charges compared to FRTB standardised approach, our analyses suggest that for a portfolio structured products, the capital savings of an internal model can be up to 70% lower, leading to RWAs that are even below the Basel 2.5 standardised approach. Firms need to understand the impact of FRTB, especially when decision to apply for model approval has not yet been taken. Firms issuing structured products can face very significant capital impacts when transitioning to the new FRTB standardised approach, which could be mitigated by the application of an IMA.

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Appendix I: Overview of the Swiss structured products market

The Swiss structured products is a 200 bio CHF market[9]. Given the current low-interest rate environment, it comes as no surprise that the most popular structured products category are the yield enhancement products, which comprises 30% of the Swiss structured products market, as illustrated in Figure 6.

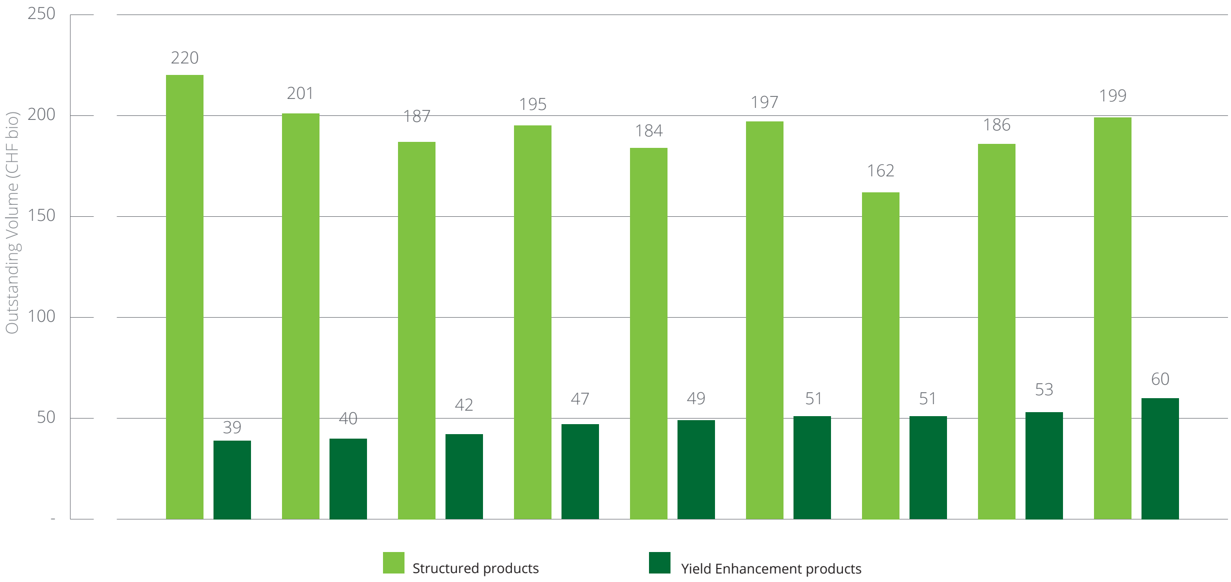

Figure 5 Outstanding Volume of Structured Products in Switzerland (bio CHF).

9. Figure at Q4 2017, data provided by SNB [8].

Appendix II: Payoff of an Autocallable Barrier Reverse Convertible

In this appendix, we give a brief background to the payoff structure of an Autocallable Barrier Reverse Convertible. We consider the following the derivative with the following specifications:

- A single underlying stock
- Maturity: 2 years
- Coupon 5% per annum, paid quarterly.
- Autocall dates at 1y, 1.25y, 1.5y and 1.75y.
- Strike level set at 100% of the initial fixing

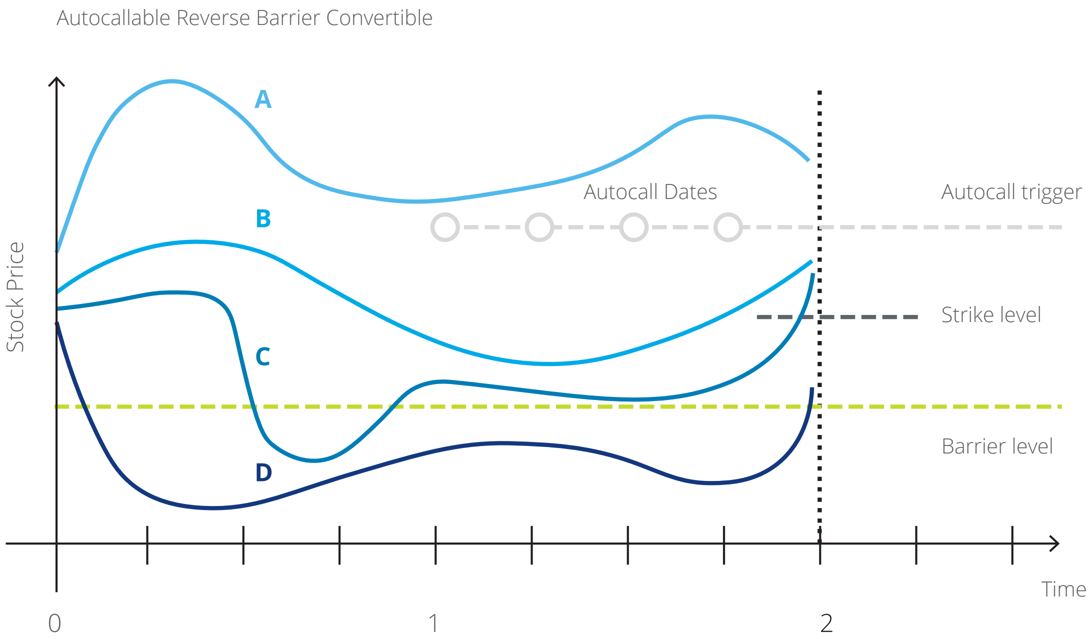

Figure 6 Example Autocallable Barrier Reverse Convertible

There are four possible scenarios for the payoff, depending on the underlying stock performance as illustrated in Figure 6 above:

Case	Description	Payout
A	The stock value exceeds the autocall trigger at one of the autocall dates (in our case at 1 year). The investor receives coupons at the first four quarters.	The contract terminates prematurely at the 1 year mark and the investor receives the contract notional.
B	No early redemption has occurred (i.e. no autocall trigger) and the stock value remains above the barrier level for the entire period of the contract.	The investor receives all coupons as well as the notional amount at the maturity of the contract.
C	No early redemption has occurred (i.e. no autocall trigger), and the stock price drops below the barrier level at some point during the contract's lifetime. At maturity, the stock is above the strike level.	The investor receives all coupons as well as the notional amount at the maturity of the contract.
D	No early redemption has occurred (i.e. no autocall trigger), and the stock price drops below the barrier level at some point during the contract's lifetime. At maturity, the stock is below the strike level.	The investor receives all coupons as well and receives physical delivery of the stock at maturity.

In the case of multiple underlying stocks, the investor carries the potential downside of the worst performing stock at maturity as soon as one of the underlying stocks breaches the barrier. In addition, the autocall feature applies to all the shocks (i.e. the derivative is autocalled provided all the underlying stocks are above the autocall trigger).

Appendix III: Sample portfolio

Within this article, we perform quantitative impact analyses on a portfolio of short autocallable barrier reverse convertibles, as a proxy of a bank's portfolio of structured products.

We assume that the portfolio is composed of five different (autocallable) barrier reverse convertibles with up to three underlying stocks. The portfolio composition is outlined in Table 1 below.

ID	Underlyings	Barrier	Strike	Mat.	Coupons	Autocallable	Notional[10] (mio CHF)
1	CS	69%	100%	2 Yrs	5% p.a. quarterly	Quarterly from year 1 with trigger 90%	-5
2	UBS	70%	100%	2 Yrs	5% p.a. quarterly	No	-3
3	CS & UBS	75%	100%	2 Yrs	5% p.a. quarterly	Quarterly from year 1 with trigger 90%	-3
4	CS & Nestle	50%	100%	2 Yrs	5% p.a. quarterly	No	-3
5	CS, UBS & Nestle	80%	100%	2 Yrs	5% p.a. quarterly	Quarterly from year 1 with trigger 90%	-3

Table 1 Sample portfolio composition

10. The notional represents the initial investment from a purchaser of the structured product. This coincides with the amount paid back to the investor at the end of the contract (excluding coupons), under the assumption that the barrier has not been breached. The negative notional here denotes a short position,i.e., the financial institution is issuing the structured product as opposed to purchasing it.

Appendix IV: Alternative risk-factor modelling approaches

Approach	Description
Driftless Brownian Motion	We assume that each underlying equity spot price S_t^i satisfies the following stochastic differential equation: $dS_t^i = \mu^i S_t^i\, dt + \sigma^i S_t^i\, dW_t^i$ Where we assume that $\mu^i = (r - d^i) = d^i.\ d^i$ represents the dividend payment and σ^i the volatility of the equity spot price. A similar stochastic process is used to simulate equity volatilities; in that case $\mu^i = 0$ and σ^i represents the volatility of volatility. Furthermore, the risk factor correlations are embedded in the multivariate (d-dimensional, equal to the number of risk factors) Wiener process W_t.
Brownian Motion with drift	In this case we assume that the drift μ^i for equity spot price S_t^i is given by $\mu^i = (\bar{\mu}^i - d^i)$, where $\bar{\mu}^i$ is the estimated drift for equity spot price S_t^i. This value is calibrated over historical Log-returns (stemming from different scenarios).
Historical Sampling	We calculate multivariate historical log-returns (based on different past scenarios). We randomly sample 10 log-returns from this dataset to simulate underlying equity spot price risk factors. Equity volatilities are simulated with the Brownian Motion approach described above.
t-Copula with historically sampled marginals	We assume that the multivariate distribution function F of d' equity spot prices is given by $F(x_1, \dots, x_{d'}) = C(F_1(x_1), \dots, F_{d'}(x_{d'})) \quad x \in \mathbb{R}^d$ Where the univariate distribution functions $F_1, \dots, F_{d'}$ are obtained from the univariate historical log-returns (based on different past scenarios).[11] C is the t-Copula with degrees of freedom $\nu = 3$ and is given by $C(u) = P(t_\nu(X_1) \leq u_1, \dots, t_\nu(X_{d'}) \leq u_{d'})$ Where X follows a multivariate t distribution $t_\nu(0, I)$. Equity volatilities are simulated with the Brownian Motion approach described above.
EWMA	Under this method we assume that the next day log returns are given by $Y = R'Z$ Where $Z = (Z_1, \dots, Z_n)$ is a vector of n iid (independent identically distributed) standard normal random variables.

11. The defined F is a multivariate distribution function with margins $F_1, \dots, F_{d'}$ as confirmed by Sklar's Theorem [4].

R is a $n \times d'$ matrix such that $Cov(Y) = R'R = C$. It is given by

$$\mathbf{R} = \sqrt{\frac{1-\lambda}{1-\lambda^n}} \begin{pmatrix} f_1(n) & f_2(n) & \cdots & f_k(n) \\ \lambda^{1/2} f_1(n-1) & \lambda^{1/2} f_2(n-1) & \cdots & \lambda^{1/2} f_k(n-1) \\ \vdots & \vdots & \cdots & \vdots \\ \lambda^{(n-1)/2} f_1(1) & \lambda^{(n-1)/2} f_2(1) & \cdots & \lambda^{(n-1)/2} f_k(1) \end{pmatrix}.$$

Where $\lambda = 0.96$ represents the decay factor and $f_j(1), \ldots, f_j(n)$ represent the last n historical return from risk factor j. This model aims to reflect the variable nature of the covariance matrix using a simple approach. The decay factor λ allows to exponentially decrease the weight given the past observations, producing simulation results mainly driven by the actual economic environment. We iterate this model, updating the R matrix after each simulated return, in order to obtain 10 simulated log-returns.

The equity spot prices returns are accurately chosen for the three different economic scenarios described above.

Equity volatilities are simulated with the Brownian Motion approach described above.

Table 2 Overview of risk factor generation approaches

Appendix V: Regulatory Background

Basel 2.5

The figures in this article stemming from the "Basel 2.5 standardised approach", correspond to the Delta Plus approach outlined in BSBS193. This approach makes use of the Greeks (delta, vega and gamma) of a derivative to measure the market risk capital requirements. The capital requirement is a sum of *Specific Risk, General Risk, Gamma Risk* and *Vega Risk*.

Specific Risk is defined as the bank's gross equity positions (sum of the absolute values of the *Delta Equivalent Notional* for each portfolio position) and *General Risk* as the bank's net equity positions (sum of the *Delta Equivalent Notional*). A separate calculation for long or short positions has to be carried out for each national market in which the bank holds equities. The *General Risk* and *Specific Risk* charge is 8%, assuming that the portfolio is not well diversified.

Gamma Risk is calculated for each underlying equity price considering a 8% price variation. Only negative net gamma impacts will be included in the capital calculation. The total gamma capital charge is given by the sum – over each underlying equity - of the absolute value of negative gamma impacts.

Vega Risk is given by the sum of vegas for all options on the same underlying, multiplied by a volatility shift of 25%.

Further details on capital risk charge calculations under Basel 2.5 can be found in [1].

FRTB Standardised Approach - SBA

The risk charge under the *Sensitivities based* method must be calculated by aggregating the following risk measures: *Delta, Vega* and *Curvature*, a risk measure that captures the incremental risk not captured by the delta risk of price changes in the value of an option. For this particular portfolio, the only relevant risk class is equity risk.

According to [2], the delta net sensitivity s_k for an underlying equity spot price k is given by

$$s_k = \frac{V_i(1.01\ EQ_k) - V_i(1.01\ EQ_k)}{0.01}$$

Equity risk sensitivities are divided into 11 buckets, according to their Market cap (large or small), economy (Emerging market and advanced economy) and sector. Each bucket has given risk weights and sensitivities correlations.

The risk position K_b for delta bucket b is given by the aggregation of weighted sensitivities within the same bucket using the prescribed correlation ρ_{kl}:

$$K_b = \sqrt{\sum_k WS_k^2 + \sum_k \sum_{k \neq l} \rho_{kl}\, WS_k WS_l}$$

In our example portfolio, we have that $WS_k = RW_k s_k$ with RW_k is equal to 50% for UBS and CS and 30% for Nestle. The intra-bucket correlation ρ_{kl} between UBS and CS is equal to 25%. The overall delta risk charge is given by

$$Delta\ risk\ charge = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c}$$

Where $S_b = \sum_k WS_k$ and $\gamma_{bc} = 15\%$[11].

In order to address the risk that correlations increase or decrease in periods of financial stress, three risk charge figures are to be calculated for each risk class, corresponding to three different scenarios on the specified values for the correlation parameters ρ_{kl} and γ_{bc}. The ultimate portfolio level risk capital charge is the largest of the three scenario-related portfolio level risk capital charges.

The FRTB standardised approach heavily penalises equity delta directional portfolios, applying risk weights up to 70% for equity spot price sensitivities. This heavy penalisation is detected in the high delta risk charge for the unhedged portfolio, as outlined in Section 6.1.2.

The Vega risk charge is calculated similarly to the Delta risk charge. The option-level vega risk sensitivity to a given risk factor is the product of the vega and the implied volatility of the option. The risk weight RW_k for UBS, CS and Nestle is ca. 77.78% and the aggregation formulas above hold for the vega risk charge calculation as well[12]. As illustrated in Section 6.1, the FRTB Standardised Approach produces much higher Vega risk charges than Basel 2.5 for this portfolio.

The *Curvature Risk* for each risk factor k is defined as the potential loss - after deducting the delta risk position – due to an upward/downward shock. *Curvature Risk* is a risk measure that aims to capture the non-linearity of the portfolio price with respect to its risk factors, i.e. the risk that is not captured by delta sensitivity. The upward/downward shock is the same as the risk weight used in the delta risk charge calculation. Gamma risk charges for negative curvature portfolios under FRTB SA are thus much higher than their corresponding gamma charges under Basel 2.5, where only a 8% shock on the underlying equity price is applied.

Our short portfolio has a positive curvature, generating a curvature risk charge equal to zero under the FRTB Standardised Approach (SA). The zero gamma risk charge for portfolio with negative curvature should not be considered as a "capital benefit" under the standardised approach. On the contrary, the Curvature Risk charge under FRTB SA removes the diversification benefit between delta and gamma charges that would be captured under an Internal Model Approach.

Delta, Gamma/Curvature and Vega risk charges under FRTB SA are expected to be much higher than Basel 2.5 for delta, vega and gamma (positive) directional portfolios.[1]

12. Note that if Delta risk charge as calculated above is negative, the bank needs to calculate the delta risk charge using the following alternative specification of $S_b = \max\left[\min\left(\sum_k WS_k, K_b\right), -K_b\right]$

13. In our example $\rho_{kl}^{(option\ maturity)} = \mathbf{1}$, since the maturities of the portfolio options are assumed to be the same. Hence, the correlations used for vega risk charge are the same as for delta risk charge.

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